Guided-Wave Characteristics of Optical Graded-Index Planar Waveguides with Metal Cladding: A Simple Analysis Method

Wei-Yu Lee, *Member, IEEE,* and Sung-Yuen Wang, *Student Member, IEEE*

***Abstract*— The guided mode characteristics of metal-clad graded-index waveguides are generally determined by complex eigenvalue equations or perturbative methods. However, these methods either require several iterations in the complex plane or are only suited to the class of waveguides whose guided field can be described analytically. We describe a method of calculating complex propagation constants for metal-clad graded-index waveguides under very general but weakly guiding conditions. The method, based on Galerkin's formalism using trigonometric basis functions, allows arbitrary inhomogeneous and complex refractive index profiles. Applications to three generally used waveguide models show our approach to be in good agreement with other analytical or numerical methods.**

I. Introduction

METAL-CLAD waveguides are important elements in integrated optics because of the great number of applications that they offer. Metallic layers frequently serve as electrodes to interface integrated optical components with other electrical or electronic circuit systems. In addition, metallic films are also useful for the protection of optical devices against stray radiation and heat dissipation. Evidently, metallic overlays will introduce additional absorption or modal loss, a feature considered to be undesirable in most applications. But recently, it was noticed that TM mode attenuation is approximately an order of magnitude greater than TE mode attenuation, and the loss dependence on the mode number is a strong function of the refractive index profile in optical waveguides [1]–[5]. These interesting characteristics have resulted in some useful applications, such as mode and polarization filtering [6]–[7]. Since total attenuation of metal-clad waveguides consists of ohmic losses in the metallic layers and scattering losses at the interfaces, graded-index slab waveguides (GISW) are expected to have lower losses than step-index slab waveguides (SISW). Thus, it is important to investigate the influence of metallic films on the propagation and attenuation characteristics of metal-clad graded-index slab waveguides.

Analysis of metal-clad waveguides is more complicated due to the fact that the refractive indices of metals of practical interest such as gold, silver, and aluminum are complex at optical frequencies, with the result that the propagation constants of all possible guided modes in waveguides become complex. However, it is essential to have a simple and fast method that can provide a detailed knowledge of modal characteristics of the metal-clad waveguides for the purpose of controlling the fabrication process and realizing certain modal properties. Many kinds of analytical or numerical methods [1]–[12] have been proposed to analyze step-index [2], [8], [9] and graded-index [3]–[5], [10]–[12], which includes linear, exponential, parabolic, and gaussian index profiles, metal-clad waveguides. The general approach is required to solve the mode complex transcendental eigenvalue equation by numerical methods such as Newton's method, which involves several iterations and great quantities of computations, especially for graded-index metal-clad waveguides. An analytical and a highly accurate numerical method to obtain the mode dispersion and attenuation of metal-clad graded-index planar waveguides is proposed by Al-Bader and Jamid [4]. While this approach both provides accurate values for the propagation constants of different modes and demonstrates explicitly how the waveguide and material parameters determine the guided field, it is only suited to the class of waveguide in which the guided field can be described analytically and the appropriate field solution often involves some special functions such as Airy, Bessel, and parabolic cylinder functions. She and Xie [5] have presented a multilayer approximation method for analyzing propagation characteristics and losses of metal-clad graded-index planar waveguides with the aid of recursion formulas. The accuracy and efficiency of this scheme depend largely on a good initial estimate of the values of the normalized propagation constants and the size of subintervals in the core region. In the above-mentioned methods, there still exists a common weakness, characterized by the impossibility to find all complex propagation constants of guided modes at one time that renders these methods laborious and cumbersome.

Recently, a very general, simple, and easy approach to use as described by Henry and Verbeek [13] has emerged as a powerful method and has been successfully implemented for many optical waveguide structures [12]–[15]. The method, often referred to as Galerkin's method [16], expands the unknown wave field in a complete set of orthogonal functions and uses this expansion to convert a linear differential equation into a set of simultaneous linear equations, i.e. the sophisticated matrix eigenvalue equation. The matrix equation will yield all the guided modes of the waveguides without any initial estimate or recurrence. Being able to yield the wave equation directly, the method does not involve subdividing the refractive

Manuscript received August 23, 1994; revised October 25, 1994.

The authors are with the Department of Electrical Engineering, Tatung Institute of Technology, Taipei, Taiwan, ROC.

IEEE Log Number 9409179.

0733–8724/95$04.00 © 1995 IEEE

index profile and additional laborious calculations. In an earlier article, Gallawa [12] used Galerkin's method with Hermite-Gaussian basis functions to calculate the complex propagation constants for nonuniform optical waveguides, which the core is lossy and the imaginary part of the refractive index is much smaller than the real part. Therefore, in this article, we try to use the different basis function, the trigonometric function, to deal with the modal characteristic problem of metal-clad waveguides, in which the lossy component is the cladding and the real and imaginary parts of the refractive index are approximately of the same order in magnitude.

The theory of this method is given in Section II. In Section III we establish the accuracy of our numerical results by comparing them with other published results, including analytical and numerical, and satisfactory results are obtained. Finally, a brief summary is given in Section IV.

II. Theory

Under the weakly guiding condition, the scalar wave equation for the TE and TM modes can be written as [4]

$$\frac{d^2\psi}{dx^2} + k_0^2\left[n^2(x) - n_e^2\right]\psi = 0 \tag{1}$$

where ψ denotes E_y for the TE case and $n(x)E_x$ for the TM case. Guided waves are assumed to propagate in the $+z$-direction, having a z-dependence and temporal as $\exp[i(\omega t - \beta z)]$. Here, $\beta = k_0 n_e$ is the propagation constant, $k_0 = 2\pi/\lambda_0$ corresponds to the free space wave number, and n_e is the unknown effective index for the modes in question. The refractive index distribution of the slab waveguides is taken to be

$$\begin{aligned} n^2(x) &= f(x) \quad x \geq 0 \\ &= n_m^2 \quad x < 0 \end{aligned} \tag{2}$$

where $f(x)$ represents the profile shape function of the graded-index film, and the complex quantity n_m^2 refers to the refractive index of the metal cladding. In the following, the formulation is limited to the TE mode. The TM mode can, of course, be treated similarly. Galerkin's method calls for the expansion of ψ in terms of a set of known orthogonal basis functions. We now express $\psi(x)$ as a sum of N trigonometric basis functions:

$$\psi(x) = \sum_{\mu=1}^{N} C_u \phi_\mu(x) \tag{3}$$

where

$$\phi_u(x) = \sqrt{\frac{2}{L_x}}\sin\left(\frac{\pi}{L_x}\mu(x - b_i)\right). \tag{4}$$

Compared with other orthogonal functions, the choice of sine functions has the advantage of using only one kind of real function. In addition, for the majority of refractive-index profiles of current interest, the integrals can be expressed in a simple closed functional form without any special restrictions, especially on the refractive index profile. Whereas, we have to emphasize that due to their unsatisfactory boundary conditions at infinity, in applying the trigonometric basis function it is necessary to restrict the waveguide structure to a finite domain large enough to ensure that the fields of guided modes of interest are nearly negligible on these artificial boundaries. Unfortunately, to our knowledge, there seem to be no general or clear-cut rules of guidance provided by any authors regarding how the optimum value of the enclosed region L_x is determined. Thus, the accuracy of the trigonometric function expansion depends critically on the choice of the enclosed region L_x and the number of terms used in the expansion N. So, we enclose the waveguide structure in the finite boundary $b_i < x < b_e$, that is, within a range of width $L_x = b_e - b_i$. In order to apply our formulation to the symmetric and asymmetric refractive index profiles, we introduce two factors, b_i and b_e, which are the x-coordinates, where the field is forced to zero. The function in (4) satisfies the orthogonality conditions

$$\frac{2}{L_x}\int_{b_i}^{b_e} \sin\left(\frac{\pi}{L_x}\mu(x - b_i)\right)\sin\left(\frac{\pi}{L_x}\nu(x - b_i)\right)dx = \delta_{\mu\nu}. \tag{5}$$

Substituting (3) into (1) and simplifying yield

$$\sum_{\mu=1}^{N} C_\mu\left[n^2(x)k_0^2 - \beta^2 - \left(\frac{\pi\mu}{L_x}\right)^2\right]\phi_\mu(x) = 0. \tag{6}$$

By multiplying (6) by $\phi_\nu(x)$, integrating over the enclosed domain, and using the orthogonality relation Equation (5), the governing linear equation is

$$\sum_{\mu=1}^{N} A_{\mu\nu}C_\mu = \beta^2\sum_{\mu=1}^{N}\delta_{\mu\nu}C_\mu \tag{7}$$

with $A_{\mu\nu}$ defined as

$$A_{\mu\nu} = -\left(\frac{\pi\mu}{L_x}\right)^2\delta_{\mu\nu} + k_0^2\int_{b_i}^{b_e} n^2(x)\phi_\mu(x)\phi_\nu(x)dx. \tag{8}$$

Obviously, the problem has been reduced to solving the well-known matrix eigenvalue (7) and (8). The eigenvalues of the square matrix A are the squared values of propagation constants β for both guided and continuum modes. The associated eigenvectors $C_{\mu\nu}$ can be substituted into (3) to approximate the field. Each of the elements of the matrix A will be complex by virtue of the metal cladding, the eigenvalues and eigenvectors are therefore complex. Finding the eigenvalues and eigenvectors of a complex matrix is simple and fast by means of modern computer routines.

III. Examples and Numerical Results

In order to assess the accuracy of our numerical method described in the previous section, we have calculated the complex propagation constant for gold-cladding planar waveguides with the asymmetric linear, truncated-parabolic, and exponential index profiles as depicted in Fig. 1. The profile shape functions are given as follows:

1) linear profile

$$\begin{aligned} f(x) &= n_1^2\left[1 - 2\Delta\left(\frac{x}{a}\right)\right] \quad 0 \leq x \leq a \\ &= n_1^2[1 - 2\Delta] = n_2^2 \quad x \geq a \end{aligned} \tag{9}$$

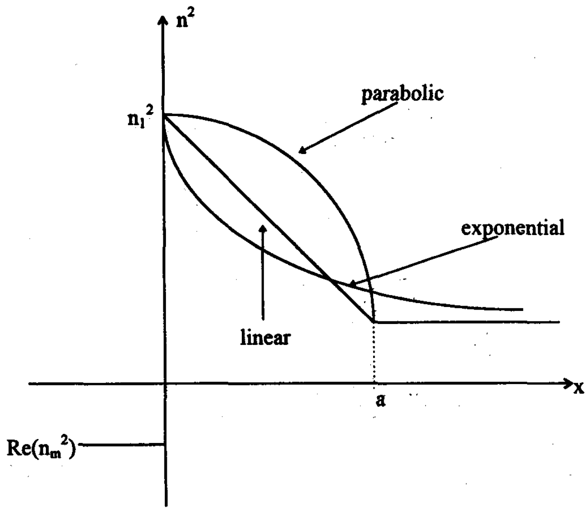


Fig. 1. Schematic diagram of index profiles for the three metal-clad waveguides considered in Section III.

TABLE I
THE COMPLEX PROPAGATION CONSTANTS OF THE LOWEST TWO TE MODES AS A FUNCTION OF THE ENCLOSED DOMAIN L_x FOR THE METAL-CLAD WAVEGUIDE WITH LINEAR INDEX PROFILE AND $a = 5\mu$m. HERE, THE NUMBER OF EXPANSION TERMS USED IN GALERKIN'S METHOD IS $N = 300$

	TE_0	TE_1
analytical [4]	14.9729 − i0.3393(−4)	14.9181 − i0.3405(−4)
numerical [4]	14.9729 − i0.3395(−4)	14.9185 − i0.3301(−4)
$[b_i, b_e] = [-1, 6]$	14.9729 − i0.3408(−4)	14.9178 − i0.3498(−4)
$[b_i, b_e] = [-1, 7]$	14.9729 − i0.3414(−4)	14.9184 − i0.3362(−4)
$[b_i, b_e] = [-1, 8]$	14.9729 − i0.3421(−4)	14.9185 − i0.3336(−4)
$[b_i, b_e] = [-1, 9]$	14.9729 − i0.3429(−4)	14.9185 − i0.3336(−4)
$[b_i, b_e] = [-1, 10]$	14.9729 − i0.3441(−4)	14.9185 − i0.3346(−4)
$[b_i, b_e] = [-3, 8]$	14.9729 − i0.3440(−4)	14.9185 − i0.3355(−4)
$[b_i, b_e] = [-5, 8]$	14.9729 − i0.3468(−4)	14.9185 − i0.3382(−4)

2) truncated-parabolic profile

$$
\begin{aligned}
f(x) &= n_1^2\left[1 - 2\Delta\left(\frac{x}{a}\right)\right] \quad 0 \le x \le a \\
&= n_1^2[1 - 2\Delta] = n_2^2 \quad x \ge a
\end{aligned}
\tag{10}
$$

3) exponential profile

$$
f(x) = n_1^2[1 - 2\Delta(1 - \exp(-x/a)] \quad x \ge 0, \tag{11}
$$

where the waveguide index parameters are $n_1^2 = 2.29592$, $n_2^2 = 2.25$, and the gold cladding $n_m^2 = -10.3 - i1.0$ at $\lambda = 0.6328$ μm. Galerkin's method may appear time-consuming and cumbersome because numerical integration is required. In fact, for trigonometric basis functions, numerical integration can be avoided in a vast majority of cases. The integrals involved in (8) can all be calculated analytically for these three profiles, as outlined in Appendix. At first glance it may appear the confusion that (A1) is not consistent with (5) and all the closed form solutions given in Appendix for the right side of (8) are functions of b_i but not functions of x. However, when explicitly substituting the limits into the equations in Appendix and after some simple mathematical operations, one quickly finds out that the expressions are exactly the same. Clearly, the numerical errors resulting from the size of a subdivision for index profiles and initial trial values are avoided because the integration of (8) is for the whole index profile without any subdivision or approximation. Although the matrix eigenvalue problem (7) can be solved in a great many ways, we prefer using the well-developed software package, PC-MATLAB$^{\text{TM}}$ (a command "eig"), to find out the desired eigenvalues and eigenvectors because it can easily deal with real, complex, or even defective matrices. Table I lists the complex propagation constants of the lowest two TE modes as a function of the enclosed domain L_x for the metal-clad waveguide with linear index profile and $a = 5$ μm. According to the data, we show how to designate the optimum size of the enclosed domain L_x. First, we choose two values arbitrarily (normally two or three multiples of the a value) for the lower boundary b_i and the upper boundary b_e as initial values. Then, we fix one side of the boundary and adjust the other side (increase or decrease), i.e., enlarging or reducing the width of the enclosed domain L_x. Following the process, we recompute the eigenmatrix A to find the desired eigenvalues in each case and determine the optimum boundary value of the adjustable side. Finally, the same procedure is used to find the boundary value of another side. Once the optimum boundary values of both sides are determined, the best size of the enclosed domain L_x is also decided. In the process of defining optimum, the reasonable accuracy between the lower-order modes and higher-order modes, especially for the imaginary part of the complex propagation constant, is an important criterion. Therefore, we select the value of [-1, 8] as the enclosed domain for the linear refractive index profile with $a = 5\mu$m. Moreover, if the accuracy of the complex propagation constant is not the only criterion, the trade-off of the accuracy between the complex propagation constant and the modal field have to be considered twice. We have used the above-mentioned rule of thumb in determining a suitable size L_x for other refractive index profiles. Tables II–VII list the analytical and numerical values, obtained by using the methods of [4], [5], and ours, of the complex propagation constants of the lowest two or three TE modes for various index profiles. In order to make a comparison with the previous published data [4], we do not list the complex propagation constants for all guided modes, but some of data is for only two modes and some of it is for three modes. According to [17], we have made some modifications about the data for the parabolic and exponential index profiles given in [4] and rewrite them in Tables IV–VII. From these tables, we observe that the agreement of the results between our method and other exact numerical methods is good in all cases. It is apparent that the expansion terms used in this article are much more than [12] and that the majority of expansion terms are used to obtain the convergence of the imaginary part of the complex propagation constants. If we take the square root of the waveguide index parameters as given in the above and obtain $n_1 = 1.5152$, $n_2 = 1.5$, and $n_m = 0.1556 - i3.2131$, it shows that we require more expansion terms to meet the slower decay rates in the region of metal cladding. Furthermore, Galerkin's method is more flexible than any other approximation method [4], [5] because

TABLE II

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH LINEAR INDEX PROFILE FOR $a = 5$ μm. THE ENCLOSED DOMAIN SIZE IS $b_i = -1$ μm, $b_e = 8$ μm, $L_x = 9$ μm

$a = 5$ μm							
mode number	β (analytical) [4]	β (numerical) [4]	β (numerical) [5]	β (ours)			
				N			
				10	100	200	300
TE_0	14.9729–i0.3393(–4)	14.9729–i0.3395(–4)	14.9729–i0.3395(–4)	14.9663–i0.9356(–4)	14.9729–i0.3891(–4)	14.9729–i0.3476(–4)	14.9729–i0.3421(–4)
TE_1	14.9181–i0.3405(–4)	14.9185–i0.3301(–4)	14.9188–i0.3301(–4)	14.9116–i0.8587(–4)	14.9184–i0.3794(–4)	14.9185–i0.3390(–4)	14.9185–i0.3336(–4)

TABLE III

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH LINEAR INDEX PROFILE FOR $a = 8$ μm. THE ENCLOSED DOMAIN SIZE IS $b_i = -1\mu$m, $b_e = 10$ μm, $L_x = 11\,\mu$m

$a = 8$ μm							
mode number	β (analytical) [4]	β (numerical) [4]	β (numerical) [5]	β (ours)			
				N			
				10	100	200	300
TE_0	14.9923–i0.2118(–4)	14.9923–i0.2119(–4)	14.9922–i0.2119(–4)	14.9854–i0.9040(–4)	14.9922–i0.3677(–4)	14.9923–i0.2206(–4)	14.9923–i0.2148(–4)
TE_1	14.9523–i0.2123(–4)	14.9523–i0.2126(–4)	14.9522–i0.2125(–4)	14.9449–i0.8871(–4)	14.9522–i0.2622(–4)	14.9523–i0.2213(–4)	14.9523–i0.2154(–4)
TE_2	14.9195–i0.2128(–4)	14.9195–i0.2110(–4)	14.9196–i0.2110(–4)	14.9116–i0.8588(–4)	14.9194–i0.2616(–4)	14.9195–i0.2208(–4)	14.9195–i0.2149(–4)

TABLE IV

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH PARABOLIC INDEX PROFILE FOR $a = 5$ μm. THE ENCLOSED DOMAIN SIZE IS $b_i = -1\mu$m, $b_e = 8$ μm, $L_x = 9\mu$m

$a = 5$ μm						
mode number	β (analytical) [4]	β (numerical) [4]	β (ours)			
			N			
			10	100	200	300
TE_0	15.0031–i0.2343(–4)	15.0031–i0.2314(–4)	14.9985–i0.7153(–4)	15.0030–i0.2654(–4)	15.0031–i0.2369(–4)	15.0031–i0.2331(–4)
TE_1	14.9465–i0.3526(–4)	14.9467–i0.3461(–4)	14.9395–i0.1005(–3)	14.9466–i0.3970(–4)	14.9467–i0.3544(–4)	14.9467–i0.3488(–4)

it can obtain any extent of accuracy in numerical results only by increasing the number of terms in the series expansion, (3).

IV. CONCLUSION

We have presented a simple and easy numerical method for to use in analyzing guided-wave characteristics of optical graded-index planar waveguides with metal cladding by means of series expansions. The complex propagation constants are found from the sophisticated matrix eigenvalue equation. Here, the integrals that appear in the matrix elements can all be solved analytically, a fact that leads to increase in the computation speed and avoidance of the numerical integration error as a result of choosing trigonometric functions as basis functions. Whereas, the need for an enclosed domain is a major disadvantage when using the trigonometric basis function. The selection of the best value of L_x is not an easy task, it is made by trial and error and case by case. However, we have provided a rule-of-thumb in dètermining the best size L_x and we see a new combination of the boundary parameters has made the rule-of-thumb more flexible. Furthermore, the numerical results show that once the suitable size L_x is selected, the sine function expansion is still accurate, compelling and viable. Since this method is inherently stationary and quite versatile, it can be easily applied to various kinds of metal-clad waveguide devices such as mode filters and polarizers of practical interest.

APPENDIX

The closed form of the last term on the right-hand side of (8) for the three index profiles can be expressed as

TABLE V

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH PARABOLIC INDEX PROFILE FOR $a = 8$ μm. THE ENCLOSED DOMAIN SIZE IS $b_i = -1\mu$m, $b_e = 10$ μm, $L_x = 11$ μm

$a = 8$ μm						
mode number	β (analytical) [4]	β (numerical) [4]	β (ours)			
			N			
			10	100	200	300
TE$_0$	15.0187–	15.0187–	15.0150–	15.0187–	15.0187–	15.0187–
	i0.1156(–4)	i0.1145(–4)	i0.5362(–4)	i0.1414(–4)	i0.1193(–4)	i0.1161(–4)
TE$_1$	14.9834–	14.9834–	14.9775–	14.9834–	14.9834–	14.9834–
	i0.1738(–4)	i0.1728(–4)	i0.7682(–4)	i0.2133(–4)	i0.1799(–4)	i0.1751(–4)
TE$_2$	14.9480–	14.9480–	14.9401–	14.9479–	14.9480–	14.9480–
	i0.2178(–4)	i0.2164(–4)	i0.9218(–4)	i0.2672(–4)	i0.2254(–4)	i0.2194(–4)

TABLE VI

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH EXPONENTIAL INDEX PROFILE FOR $a = 3\mu$m. THE ENCLOSED DOMAIN SIZE IS $b_i = -1$ μm, $b_e = 14$ μm, $L_x = 15$ μm.

$a = 3$ μm						
mode number	β (analytical) [4]	β (numerical) [4]	β (ours)			
			N			
			10	100	200	300
TE$_0$	14.9615–	14.9615–	14.9460–	14.9613–	14.9615–	14.9615–
	i0.3296(–4)	i0.3369(–4)	i0.4497(–3)	i0.4639(–4)	i0.3682(–4)	i0.3472(–4)
TE$_1$	14.9208–	14.9208–	14.9115–	14.9207–	14.9208–	14.9208–
	i0.1866(–4)	i0.1924(–4)	i0.2573(–3)	i0.2647(–4)	i0.2102(–4)	i0.1982(–4)
TE$_2$	14.9014–	14.9014–	14.8965–	14.9013–	14.9013–	14.9013–
	i0.9047(–5)	i0.9296(–5)	i0.1355(–4)	i0.1319(–4)	i0.1048(–4)	i0.9884(–5)

TABLE VII

ANALYTICAL AND NUMERICAL VALUES OBTAINED USING DIFFERENT APPROXIMATIONS OF THE COMPLEX PROPAGATION CONSTANTS OF TE MODES FOR THE WAVEGUIDE WITH EXPONENTIAL INDEX PROFILE FOR $a = 5$ μm. THE ENCLOSED DOMAIN SIZE IS $b_i = -1$ μm, $b_e = 13$ μm, $L_x = 14$ μm.

$a = 5$ μm							
mode number	β (analytical) [4]	β (numerical) [4]	β (numerical) [5]	β (ours)			
				N			
				10	100	200	300
TE$_0$	14.9820–	14.9820–	14.9829–	14.9710–	14.9818–	14.9819–	14.9820–
	i0.2357(–4)	i0.2385(–4)	i0.2312(–4)	i0.2682(–3)	i0.3285(–4)	i0.2573(–4)	i0.2449(–4)
TE$_1$	14.9459–	14.9459–	14.9466–	14.9373–	14.9458–	14.9459–	14.9459–
	i0.1684(–4)	i0.1714(–4)	i0.3460(–4)	i0.2004(–3)	i0.2361(–4)	i0.1849(–4)	i0.1760(–4)
TE$_2$	14.9236–	14.9236–		14.9171–	14.9235–	14.9235–	14.9236–
	i0.1194(–4)	i0.1221(–4)		i0.1480(–3)	i0.1689(–4)	i0.1322(–4)	i0.1259(–4)

1) For a linear profile:

$$\int \phi_\mu(x)\phi_\nu(x)dx = \frac{2}{L_x} \cdot \begin{cases} (x-b_i)/2 - \sin[2m(x-b_i)]/4m & m = n \\ \sin[(m+n)(b_i - x)]/[2(m+n)] \\ \quad - \sin[(m-n)(b_i-x)]/[2(m-n)] & m \neq n \end{cases} \tag{A1}$$

and

$$\int x \cdot \phi_\mu(x) \cdot \phi_\nu(x)dx = \frac{2}{L_x}$$

$$\cdot \begin{cases} x^2/4 - \cos[2m(b_i - x)]/8m^2 \\ \quad + x\sin[2m(b_i-x)]/4m & m = n \\ -\cos[(m+n)(b_i-x)]/2(m+n)^2 \\ \quad + \cos[(m-n)(b_i-x)]/2(m-n)^2 \\ \quad + x\sin[(m+n)(b_i-x)]/2[(m+n) \\ \quad - x\sin[(m-n)(b_i-x)]/2(m-n) & m \neq n \end{cases} \tag{A2}$$

2) For truncated-parabolic profile:

$$\int x^2\phi_\mu(x) \cdot \phi_\nu(x)dx = \frac{2}{L_x}$$

$$\cdot \begin{cases} x^3/6 - x\cos[2m(b_i - x)]/4m^2 \\ \quad - \sin[2m(b_i - x)]/8m^3 \\ \quad + x^2 \sin[2m(b_i - x)]/4m & m = n \\ -x\cos[(m+n)(b_i - x)]/(m+n)^2 \\ \quad + x\cos[(m-n)(b_i - x)]/(m-n)^2 \\ \quad - \sin[(m+n)(b_i - x)]/(m+n)^3 \\ \quad + x^2 \sin[(m+n)(b_i - x)]/2(m+n) \\ \quad + \sin[(m-n)(b_i - x)]/(m-n)^3 \\ \quad - x^2 \sin[(m-n)(b_i - x)]/[2(m-n)] & m \neq n \end{cases} \tag{A3}$$

3) For exponential profile:

$$\int \exp(-x/a) \cdot \phi_\mu(x) \cdot \phi_\nu(x) dx = \frac{2}{L_x}$$

$$\cdot \begin{cases} \exp(-x/a)\{-a/2 + [a\cos[2m(b_i - x)] \\ \quad + 2a^2 m \sin[2m(b_i - x)]] \\ \quad [2(1 + 4a^2 m^2)]\} & m = n \\ a\exp(-x/a)\{\cos[(m+n)(b_i - x)]/ \\ \quad [2(1 + a^2(m+n)^2)] \\ \quad - \cos[(m-n)(b_i - x)]/[2(1 + a^2(m-n)^2)] \\ \quad + a(m+n) \\ \sin[(m+n)(b_i - x)]/[2(1 + a^2(m+n)^2)] \\ \quad - a(m-n) \\ \sin[(m-n)(b_i - x)]/[2(1 + a^2(m-n)^2)]\} & m \neq n \end{cases} \tag{A4}$$

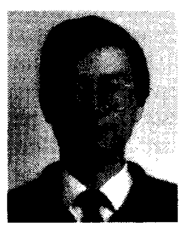
where $m = \dfrac{\pi\mu}{L_x}$, $n = \dfrac{\pi\nu}{L_x}$.

ACKNOWLEDGMENT

The authors are indebted to Dr. R. L. Gallawa for his many stimulating comments and suggestions and unknown reviewers for a critical reading of the manuscript. The authors are grateful to Professor Hsin-Hsiung Lin in Tatung Institute of Technology for providing many valuable English grammar suggestions and editorial assistance. We also thank Ming-Dong Wu in National Taiwan University for many helpful discussions and assistance, and Wen-Bin Lin for providing his personal computer.

REFERENCES

[1] S. J. Al-Bader and H. A. Jamid, "Comparison of absorption loss in metal-clad optical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 310–314, 1986.
[2] E. M. Garmire and H. Stoll, "Propagation losses in metal-film-substrate optical waveguides," *IEEE J. Quantum Electron.*, vol. QE-8, pp. 763–766, 1972.
[3] S. J. Al-Bader and H. A. Jamid, "Guided mode characteristics of metal-clad planar optical waveguides produced by diffusion," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 587–591, 1987.
[4] S. J. Al-Bader and H. A. Jamid, "Guided wave characteristics of metal-clad graded-index planar optical waveguides: analytical approach," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 539–544, 1987.
[5] S. X. She and F. C. Xie, "Propagation characteristics and loss of metal-clad graded index optical waveguides," *Optics Commun.*, vol. 103, pp. 365–369, 1993.
[6] H. A. Jamid and S. J. Al-Bader, "TM-pass polarizer using metal-clad waveguide with high index buffer layer," *Electron. Lett.*, vol. 24, pp. 229–230, 1988.
[7] S. X. She, "Metal-clad optical polarizer: perturbation analysis," *Optics Commun.*, vol. 105, pp. 175–177, 1994.
[8] A. Reisinger, "Characteristics of optical guided modes in lossy waveguides," *Appl. Opt.*, vol. 12, pp. 1015–1025, 1973.
[9] I. P. Kaminow, W. L. Mammel, and H. P. Weber, "Metal-clad waveguides: Analytical and experimental study," *Appl. Opt.*, vol. 13, pp. 396–405, 1974.
[10] M. Masuda, A. Tanji, Y. Ando, and J. Koyama, "Propagation losses of guided modes in an optical graded-index slab waveguide with metal cladding," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 773–776, 1977.
[11] T. Findakly and C. L. Chen, "Diffused optical waveguides with exponential profile: Effects of metal-clad and dielectric overlay," *Appl. Opt.*, vol. 17, pp. 469–474, 1978.
[12] R. L. Gallawa, "Complex propagation constants for nonuniform optical waveguides: Calculations," *Microwave Opt. Technol. Lett.*, vol. 6, pp. 490–493, 1993.
[13] C. H. Henry and B. H. Verbeek, "Solution of the scalar wave equation for arbitrarily shaped dielectric waveguides by two-dimensional Fourier analysis," *J. Lightwave Technol.*, vol. LT-7, pp. 308–313, 1989.
[14] S. Y. Wang and W. Y. Lee, "Analyzing integrated-optical inhomogeneous planar waveguides by Galerkin's method: a detailed comparison of two different basis functions," *Fiber Integrated Opt.*, vol. 13, pp. 407–420, 1994.
[15] D. Marcuse, "Solution of the vector wave equation for general dielectric waveguides by the Galerkin method," *IEEE J. Quantum Electron.*, vol. QE-28, pp. 459–465, 1992.
[16] S. G. Mikhlin and K. L. Smolitskiy, *Approximate Method for Solution of Differential and Integral Equations.* New York: Elsevier, 1967.
[17] W. Y. Lee and S. Y. Wang, "Comments on "Guided wave characteristics of metal-clad graded-index planar optical waveguides: Analytical approach," submitted to *IEEE J. Quantum Electron.*



Wei-Yu Lee (M'89) was born in Taipei, Taiwan, ROC, on February 22, 1957. He received the B.S. in electrical engineering from Tatung Institute of Technology, Taipei, Taiwan, ROC, in 1979, and the M.S. and Ph.D. degrees in electrical engineering from National Taiwan University in 1983 and 1988, respectively.

From 1983 to 1988, he served as a teaching assistant in the Department of Electrical Engineering, National Taiwan University. Since 1988, he has been with the Department of Electrical Engineering, Tatung Institute of Technology as an associate professor. His research interests include semiconductor devices and organic and inorganic integrated optics.

Sung-Yuen Wang (S'90) was born in Tainan, Taiwan, ROC, on October 28, 1967. He received the B.S. degree in electrical engineering from Tatung Institute of Technology, Taipei, Taiwan, ROC in 1989. Currently, he is pursuing his Ph.D. degree in electrical engineering at Tatung Institute of Technology. His current research interests include guided-wave optics and organic integrated optics.